How Bad is the Merger Paradox?

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Abstract. The merger paradox is a classic, counter-intuitive result from the literature of Industrial Organization saying that merging firms typically experience a decline in their overall profit compared to their total pre-merger profit. This phenomenon is more striking in small oligopolistic markets, where mergers increase market concentration and may hence trigger a substantial increase in prices. In this paper, we investigate the severity of the merger paradox in Cournot oligopoly markets. Namely, we study the worst-case magnitude of this profit loss in quantity-setting market games. We consider convex, asymmetric production costs for the firms, and we show that the profit loss can be substantial even in small markets. That is, two merging firms can lose half of their pre-merger profit, but no more than half in markets with concave demand functions. On the positive side, we show that in markets with affine demand two firms can never lose more than 1/9 of their profit when merging, and this bound is tight. We also study the asymptotic loss in larger markets, where it is easy to show that the profit loss can be arbitrarily large when multiple firms merge; we give bounds that characterize the profit loss from a merger as a function of the market size and the number of merging firms.

Keywords: Cournot Oligopoly · Approximation · Nash Equilibrium · Mergers and Acquisitions · Industrial Organization · Market Structure

1 Introduction

Evaluating the benefits from mergers of firms is an important challenge both for managers and for regulators worldwide. A merger of firms is a costly, often irreversible, process and therefore managers should make sure that the merger is indeed a profitable business move. Mergers increase concentration in markets, and antitrust authorities all over the world try to predict the effect of such mergers on the market power of firms and on the consumer surplus. This paper belongs to a line of work that aims to improve our understanding of the potential outcome of mergers. In this paper, we take a worst-case approach to this problem. We do not assume any probabilistic assumptions or knowledge of prior distributions. We give results that hold for all possible demand and cost functions under standard assumptions. Several recent papers applied algorithmic thinking for analyzing competition in markets, see, e.g., [20, 24, 3, 5, 2, 1, 22, 33].

Finding the right model for portraying competition in markets has always been one of the greatest challenges in economics. When competition is modeled as a price-setting game (á la Bertrand [4]), it is well known that the outcome tends to be highly competitive and that mergers are beneficial both for the merging firms and for the other firms (e.g., [8]). Another popular way to model competition is by quantity-setting games, where the basic model is known as the Cournot oligopoly model ([6]) – which is arguably the most influential model of competition, both for theoretical and applied modeling (e.g., [19]). Cournot models are especially useful in homogeneous-good environments where quantity decisions are set in advance, like in the airline industry, computer chips, cars, etc. Cournot models markets as a game, where firms simultaneously determine the output they produce, and the price is determined by the total output of all firms via the demand curve. An equilibrium in the Cournot model is a Nash equilibrium – a situation where no firm would benefit from unilaterally changing its production level. The solution to the Cournot model has many desirable properties, e.g., it lies somewhere between the Cartelistic and the competitive outcome and the equilibrium price decreases as the number of competitors increases [23].

Our paper focuses on the analysis of mergers in the Cournot oligopoly model. When some of the firms in the market merge, this creates a new game where the merged firm determines its production level as one entity. The benefits for the merging firms are quite straightforward – mergers create more concentrated markets with higher prices for consumers. Moreover, firms produce more efficiently in the merged firm and thus their manufacturing costs drop.[3] This intuition makes the following classic result, called the *Merger Paradox*, quite surprising: According to the Merger Paradox, not only that there exist scenarios where the profit of the merged firm will be smaller than the total profit of the firms before merger, this is actually the typical case. Beyond the theoretical analysis (see references below), the merger paradox was also tested empirically, where it was shown that the market values of some companies declined after merging (see, e.g., [13, 17] and the references within).

We will show settings where two merged firms can lose up to half of their profit by deciding to merge, but our main result shows that for affine demand curves (and convex, asymmetric cost functions) the profit loss of 1/9 is the worst example.

The intuition for the merger paradox is quite easy in large markets. If we have a market with 1000 symmetric firms and two of them merge, the effect of the merger on the concentration in the market is negligible, but the two firms that had a pre-merger market share of $\frac{1}{1000}$ *each*, now hold a market share of $\frac{1}{999}$ *together*! In fact, in Cournot's original setting with many firms, at least 80% of the firms should be included in the merger to guarantee its profitability (e.g., [26]). It is hard, however, to find such an intuition for smaller markets. If two

[3] Examples of profitable mergers are relatively common. For instance, consider a market with three firms, where one of the firms plays a small role in it, in terms of its market share – a merger of the other two results in a market which relatively resembles a monopoly, hence being beneficial for the merging firms.

out of three firms merge, one may expect that after merging, the two firms in the market will have a much stronger market power that will lead to a substantial increase in prices. Still, even mergers of two out of three symmetric firms will often lead to a profit loss for the merged firm.

The merger paradox builds on some other subtleties, and it seems to be confusing at first glance. For example, how can the merging firms lose when they can still produce at exactly the same pre-merger level? The answer is that the merger creates a new game with a new incentive structure and this production level will no longer be a best response to the quantity produced by the other, unmerged, firms; the merged company will actually have the incentive to cut back the produced quantity. The other firms understand the new incentive structure as well, and the post-merger equilibrium may be completely different.[4] In addition, there is evidence in the literature that other factors which are not taken into account in the Cournot model make mergers more profitable. One such factor is *Coordinated Effects* (see the survey [9]), saying that mergers change the market structure in a way that makes it easier for the remaining firms to collude, and makes the collusion more effective, either legally or illegally.

In this paper we quantify the magnitude of the merger paradox. We give a worst-case analysis of the potential loss from mergers, and we give tight bounds for the maximal possible loss. For the most popular and important mergers, of two firms, we show that the this profit loss can be high in general, but relatively-mild when the demand functions are affine.

1.1 Our Results

We consider the classic Cournot model of n competing firms that produce and sell a single, divisible, homogeneous good. Each firm F_i has a cost function $C_i(\cdot)$ for producing identical units of the good. We assume the standard assumptions that the cost functions are non-decreasing and convex – that is, that the marginal production cost is positive and increasing. When two firms merge, they can produce in the factory of the first firm (i.e., according to C_1), or in the factory of the second firm (C_2), or split the production between the two factories in order to reduce costs (as in standard cartel models). The market demand is represented by a demand function $P(\cdot)$. We give results for two families of demand functions: concave demand and affine demand. Concave demand is a general sufficient condition for the existence and uniqueness of equilibrium in the Cournot market ([30]), and thus analyzing the Cournot model without this assumption presumably requires a different approach than the one we take in our analysis. The case of affine demand functions is a special case which is well-studied in the literature (see e.g., [28, 26, 29, 11]).

As mentioned earlier, the merger paradox is mostly interesting for small markets. Therefore, our main results are given for mergers of two out of three

[4] Sometimes merged firms choose, or are even forced, to create a "firewall" between the merged divisions – however, this custom is infrequently used and it is hard to be maintained in the long run [15].

firms. We also discuss their extensions to larger markets. Our first result shows that when two firms merge, there are settings where they lose half of their pre-merger profit, even in small markets. We also show that for all markets with concave demand functions such firms lose at most half.

Theorem 1: (Informal) *In a market with concave demand and three firms with convex costs, two merging firms can lose up to half of their pre-merger profits. This bound is tight.*

We extend the above results to larger markets. We show that if k out of n firms merge, they lose at most a factor of $1 - \frac{1}{k}$ of their pre-merger profit, and we show that this bound is asymptotically tight in large markets, from which it follows that the profit loss from mergers can be arbitrarily large. As mentioned, in large markets it is quite straightforward that the profit loss of two merging firms is almost half (see the above intuition about the merger of 2 out of 1000 symmetric firms); the above result proves that this holds even in small markets.

Our main result is for markets with affine demand, where we show that the profit loss is mild for all convex cost functions:

Theorem 2: (Informal) *In a market with affine demand and three firms with convex costs, two merging firms can lose up to $\frac{1}{9}$ of their pre-merger profits. This bound is tight.*

The asymptotic bound for merging k out of n firms which was mentioned above also holds for the case of affine demand, hence even with affine demand the profit loss from mergers can be arbitrarily large.

We will now describe the technical gist of our main result. The simplest form of the merger paradox concerns mergers with no cost synergies, that is, situations where the merging firms have symmetric, constant marginal costs (as in the above example). In this simple case, the firms do not save any production costs due to the merger. Thus, it crystallizes the tradeoff between the increased market concentration on one hand and the decrease in the relative market weight of the merged firms on the other hand (e.g., the transition from 2 out of 3 firms to 1 of 2). We first show that in this simple case, two merging firms will always lose $1/9$ of their profit from merging (ignoring some degenerate cases). If the firms have *asymmetric* constant marginal costs (i.e., linear cost functions), then by merging they can produce at the factory with the lower marginal costs and therefore gain additional benefits from merging. One would hope that we can directly reduce any profile of cost-functions to the linear-cost case, and show this way that the worst-case loss is indeed $1/9$ for all convex costs. Unfortunately, the treatment of some of the cases shows that this simple reduction does not work. We bypass this problem by reducing it to the analysis of a linear-cost market that relaxes the assumption of equilibrium production levels.

We conclude with a new paradoxical example regarding the Cournot model, which unlike the above results is unrelated to mergers. This example considers technological improvements that reduce production costs of the firms. It is known ([30]) that if the technology of one firm is improved, such that its marginal cost for every additional unit decreases, then the profit of this firm must increase in the new Cournot equilibrium. However, we note that if the costs of two firms in the market improved as above, it might be the case that the total profit of the two firms in the new Cournot solution *decreases*. We show that this profit loss is mild for linear costs, but a further analysis of this surprising phenomenon is left for future work. Due to space limitations, this result appears in the full version of our paper.

1.2 Related Work

The merger paradox is originated in the work by Salant, Switzer and Reynolds [26], who showed how the merger paradox appears in Cournot's original setting from [6]. [26] showed that in all markets with linear costs, any merger of less than 80% of the firms will lead to unprofitable mergers. [16] modeled the multi-firm merger decision as a multi-stage game, and identified its sub-game perfect equilibria where the owner of the group of firms may be better off by letting some of the firms compete against the others. [21] showed how allowing the merged firm to be a market (Stackelberg) leader can mitigate the merger paradox. More papers devised models that yield beneficial mergers (e.g., [7, 10]).

Probably the closest papers to our work are papers by Tsitsiklis and Xu [31, 32]. They compare the Cournot outcome to the optimal outcome both in terms of profit [31] and social efficiency [32]. Our approach in this paper is similar to [31, 32] as we also take a worst-case approximation approach and we prove our main result via a reduction to the case of linear cost functions. However, [31, 32] compare the Cournot outcome to the post-merger outcome only in the case of a complete merger – where all the market participants merge to a monopoly (and clearly improve their profit). Also, our results require different techniques. As an example, we reduce the general case to a generalized variant of markets with linear costs, as attempts of applying a straightforward reduction prove futile. As another example, we deploy a concrete computation of the equilibria in the reduced markets. Since many comparative statics results regarding a monopoly formation do not hold in our case – the use of these computations is crucial for our analysis. An earlier paper by Johari and Tsitsiklis [14] showed the Cournot outcome achieves at least 2/3 of the maximal social efficiency for markets with concave demand using the Price of Anarchy approach [12, 18, 25].[5]

2 Model

We consider a game with n firms, denoted by $F_1, \dots, F_n$. Each firm F_i chooses a quantity $x_i \in [0, \infty)$ to be supplied by it. The *inverse demand function*, denoted

[5] We note that our framework is fundamentally different from Price-of-Anarchy models. Price of Anarchy analysis compares an equilibrium outcome to some unrealistic optimal solution. Our approach is to compare two practical alternatives for decision makers: markets with or without mergers. For managers, who need to decide whether to merge or not, and for regulators who need to approve mergers – these are two realistic situations they need to carefully understand.

by $P(X)$, represents the price per unit the consumers are willing to pay, given that the total production of the firms is $X = \sum_{j=1}^{n} x_j$. The *cost function* of F_i, denoted $C_i(x)$, represents its cost for producing quantity x. The *profit* of F_i is

$$\Pi_i(x_1, \ldots, x_n) = P(x_1 + \ldots + x_n) \cdot x_i - C_i(x_i) \tag{1}$$

The Nash equilibrium point of such game is termed a *Cournot equilibrium point*. That is, a Cournot equilibrium is a vector $x = (x_1, \ldots, x_n)$ such that

$$\forall i \in [n], \forall x_i' \in [0, \infty) \quad \Pi_i(x_i, x_{-i}) \geq \Pi_i(x_i', x_{-i}) \tag{2}$$

where we use the standard notation x_{-i} to denote the vector of strategies of all firms except for F_i.

We provide two assumptions on $P(\cdot)$ and the functions $C_i(\cdot)$, that will be used throughout this paper:

Assumption 1: The function $P(\cdot)$ is differentiable, strictly decreasing and concave on the part where it is positive. Moreover, it is non-negative with $P(0) > 0$.
Assumption 2: The functions $C_i(\cdot)$ are all continuously differentiable, non-decreasing and convex. Moreover, they all maintain that $C_i(0) = 0$.

For brevity, we refer to Assumption 1 as the "concave demand function" assumption and similarly to Assumption 2 as the "convex cost functions" assumption, although the rest of the details are also in force. Note that concavity of the inverse demand function implies the concavity of its inverse, namely the demand function, which we also occasionally refer to instead. The convexity assumption imposed on the cost functions is rather standard in economics literature, and follows from the law of diminishing returns, which states that increasing a factor of production (e.g., labor, capital) by one unit, ceteris paribus, results in lower output per incremental input unit [27]. The concavity assumption on $P(\cdot)$, in turn, ensures the existence and uniqueness of the equilibrium point, as well as some other properties we mention later. While it only provides a sufficient condition for uniqueness, there is a rich and well established literature with various results, many of which are used in the sequel, that use this assumption. Some of our results and our analysis are particularly concerned with the special case of affine demand functions. [6]

The purpose of this paper is to compare the profits of firms in equilibrium in two states, before some of the firms merge and afterwards. We consider a merger of any subset of firms in our analysis, and provide bounds on the ratio of those two profits. This motivates the assumption by which $\forall i \; C_i(0) = 0$, that ensures we avoid comparing positive profits (or costs) to negative ones, resulting in negative bounds.

Assuming that the firms $F_1, \ldots, F_k$ merged, we denote the merged firm by $F_{1,\ldots,k}$. We think of that firm as having multiple factories, such that each factory i corresponds to the firm F_i before merging. Therefore, a strategy $\widetilde{x}_{1,\ldots,k}$ of the

[6] Note that affine functions also have negative values. The use of such functions is still legitimate, though, as the firms do not produce quantities for which the functions are negative, as we show in our proofs.

merged firm can be represented by a (not necessarily unique) vector of quantities $(\widetilde{x}_1, \ldots, \widetilde{x}_k)$, such that $\widetilde{x}_i$ is the quantity produced by factory i, and interpreted as $\widetilde{x}_{1,\ldots,k} = \widetilde{x}_1 + \ldots + \widetilde{x}_k$. We define the cost function of the merged firm as

$$\widetilde{C}_{1,\ldots,k}(x) = min \left\{ \sum_{j=1}^{k} C_j(x_j) \mid \sum_{j=1}^{k} x_j = x,\ \forall j \in [k]\ x_j \geq 0 \right\} \tag{3}$$

This function is well defined as the minimum is taken on a continuous function over a compact set.

Further notations used throughout this paper include the following. Any market M with n firms has two states, as already mentioned, before any merging occurred and afterwards. Its Cournot equilibrium (which exists and is unique under Assumption 1 and Assumption 2, as we mention later), before any merging took place, is denoted by $x^M = (x_1^M, \ldots, x_n^M)$. The profit and cost functions are denoted with the superscript M. For example, $\Pi_2^M(x^M)$ is the profit of F_2 in equilibrium before merging. For brevity, we also denote the profit of F_i in equilibrium by Π_i^M instead of $\Pi_i^M(x^M)$, and omit the superscript M when it is clear. The Cournot equilibrium in M in its second state, after $F_1, \ldots, F_k$ merged for some predefined k, is denoted by $\widetilde{x}^M = (\widetilde{x}_{1,\ldots,k}^M, \widetilde{x}_{k+1}^M, \ldots, \widetilde{x}_n^M)$. The profit and cost functions are denoted similarly, with the same shorthand notations applied. For example, $\widetilde{\Pi}_{k+1}^M = \widetilde{\Pi}_{k+1}^M(\widetilde{x}^M)$ is the profit of F_{k+1} in equilibrium after $F_1, \ldots, F_k$ merged. When we do not mention with which of the two states an equilibrium is affiliated, the pre-merger state or the post-merger state, we refer to the former.

With those notations, we say that $F_1, \ldots, F_k$ lose a fraction of η of their total pre-merge profits (assuming they lose profits at all) if $\widetilde{\Pi}_{1,\ldots,k} = (1-\eta) \cdot (\Pi_1 + \ldots + \Pi_k)$. As we are normally interested in cases that cause loss of profits by merging, the pre-merger profits are positive in those, i.e., $\Pi_1 + \ldots + \Pi_k > 0$. Therefore, the *fraction of loss* is merely $\eta = 1 - \frac{\widetilde{\Pi}_{1,\ldots,k}}{\Pi_1 + \ldots + \Pi_k}$.

2.1 Known Properties of Cournot Markets

In this subsection, we mention a few known and existing properties from the literature that are guaranteed to hold in markets satisfying the aforementioned assumptions. These properties are presented in the following lemmas, which all assume Assumption 1 and Assumption 2. The first two of those lemmas are well known in economics literature and textbooks (see, e.g., [30, 31] and the references within). The other three are from a work by Szidarowsky and Yakowitz [30].

Lemma 1. *The following are necessary and sufficient conditions for a vector $x = (x_1, \ldots, x_n)$ to be a Cournot equilibrium, which exists and is unique in markets with a concave demand and convex costs. For each $i \in [n]$:*

$$\text{If } \ x_i > 0: \quad C_i'(x_i) = P(x_1 + \ldots + x_n) + x_i \cdot P'(x_1 + \ldots + x_n) \tag{4}$$

$$\text{If } \ x_i = 0: \quad C_i'(x_i) \geq P(x_1 + \ldots + x_n) + x_i \cdot P'(x_1 + \ldots + x_n) \tag{5}$$

If all firms produce according to Equation (4), the corresponding equilibrium is called an *internal Cournot equilibrium*. If, however, some firm violates it, we say that the equilibrium is a *corner solution* for that firm.

Lemma 2. *If firms $F_1, .., F_k$ merge in a market with a concave demand and convex costs, for some $k \leq n$, the function $\widetilde{C}_{1,..,k}(\cdot)$ maintains Assumption 2 in the post-merger market. Moreover, if each factory F_i produces a quantity of $\widetilde{x}_i$ units after merging, then for each $i \in [k]$:*

$$\text{If } \widetilde{x}_i > 0 : \quad C_i'(\widetilde{x}_i) = \widetilde{C}'_{1,..,k}(\widetilde{x}_1 + \ldots + \widetilde{x}_k) \tag{6}$$

$$\text{If } \widetilde{x}_i = 0 : \quad C_i'(\widetilde{x}_i) \geq \widetilde{C}'_{1,..,k}(\widetilde{x}_1 + \ldots + \widetilde{x}_k) \tag{7}$$

Lemma 3. *If some firms merge, each of the other firms produces in equilibrium, in markets with a concave demand and convex costs, at least the amount it produced before that merging. That is, if $x = (x_1, \ldots, x_n)$ is the Cournot equilibrium before $F_1, \ldots, F_k$ merged, for some $k \leq n$, and $\widetilde{x} = (\widetilde{x}_{1,\ldots,k}, \widetilde{x}_{k+1}, \ldots, \widetilde{x}_n)$ is the equilibrium afterwards, then $\widetilde{x}_i \geq x_i$ for each $i \geq k+1$.*

Lemma 4. *If some firm F_i improves its cost function $C_i(\cdot)$ to $\widehat{C}_i(\cdot)$, in the sense that for each $x \geq 0$ it holds that $C_i'(x) \geq \widehat{C}_i'(x)$, then both F_i's production level and its profit in equilibrium can only increase in markets with a concave demand and convex costs. The production level and the profit of each of the other firms can only decrease in that case.*

Another important lemma proved in [30] states that if some firm leaves the market, the other firms can only benefit, and when some firm joins the market, the other firms' profits can only decrease. To emphasize the connections between the lemmas in this section, we provide a new proof for this lemma, which is shorter, and is based on a reduction from the analysis of omitting a firm to the analysis of replacing an existing firm's cost function. This new proof can be found in the full version of our paper.

Lemma 5. *If some firm F_i leaves the market, then the profit of each of the other firms in equilibrium is at least as it was before F_i left, in markets with a concave demand and convex costs. On the other hand, if some firm joins the market, the profit of the others can only decrease in equilibrium.*

3 Markets with Concave Demand

Our main result in this section is that the merger paradox may be severe when two firms merge, namely, two firms can lose half of their profit by merging. While this is intuitive in large markets, we show that this may happen even when two out of three firms merge. On the other hand, we show that this is indeed the worst case for markets with concave demand; in such markets, two merging firms will lose *at most half* of their pre-merger profit. We also give asymptotic results for mergers of k firms out of n, and in particular we show that in some markets, the losses incurred by merging can be arbitrarily high.

We start by proving the following lemma that shows that the profit of the merged firm is at least the profit of each of the merging firms, prior to merging. The lemma is a key ingredient in our analysis. We state and prove the lemma for the general case of k merging firms out of a total of n firms in a market with concave demand.

Lemma 6. *Consider a market with a concave demand function and $n \geq 3$ firms, each having a convex cost function. When $F_1, \ldots, F_k$ merge, for some $k \leq n$, the profit of the merged firm in equilibrium is at least the profit of F_i in equilibrium before merging, for any $i \in [k]$.*

Proof. Denote the market by M_1. By Lemma 2, M_1 in its post-merger state maintains Assumption 1 and Assumption 2. So, by Lemma 1, the Cournot equilibrium in M_1 before $F_1, \ldots, F_k$ merge exists and is unique, and so is the Cournot equilibrium in M_1 afterwards. We show that

$$\widetilde{\Pi}^{M_1}_{1,\ldots,k} \geq max\left\{\Pi^{M_1}_1, \Pi^{M_1}_2, \ldots, \Pi^{M_1}_k\right\} \tag{8}$$

Let $i \in [k]$. Denote by M_2 the market in which F_i replaces its cost function by $\widetilde{C}^{M_1}_{1,\ldots,k}(\cdot)$. It has a unique Cournot equilibrium. We show the inequality $\Pi^{M_2}_i \geq \Pi^{M_1}_i$. Consider the factories $F_1, \ldots, F_k$ that constitute the merged firm $F_{1,\ldots k}$. By Lemma 2, for each non-negative $x_1, \ldots, x_k$ and x such that $x = \sum_{i=1}^k x_i$, and such that each factory F_i produces x_i units, each of $C'^{M_1}_1(x_1), \ldots, C'^{M_1}_k(x_k)$ is at least $\widetilde{C}'^{M_1}_{1,\ldots,k}(x)$. Since $C^{M_1}_i(\cdot)$ is convex, its slope is increasing in its input, so $x \geq x_i$ implies that $C'^{M_1}_i(x) \geq C'^{M_1}_i(x_i) \geq \widetilde{C}'^{M_1}_{1,\ldots,k}(x)$. So, by Lemma 4, it indeed follows that

$$\Pi^{M_2}_i \geq \Pi^{M_1}_i \tag{9}$$

Now, denote by M_3 the market in which we omit each F_j for $j \in [k] \setminus \{i\}$ from M_2. It also has a unique Cournot equilibrium. Thus, by Lemma 5:

$$\Pi^{M_3}_i \geq \Pi^{M_2}_i \tag{10}$$

Note that M_1, in its state after merging, is exactly M_3 (before any merging occurred). Combining Inequality (9) and Inequality (10), we obtain that

$$\widetilde{\Pi}^{M_1}_{1,\ldots,k} = \Pi^{M_3}_i \geq \Pi^{M_1}_i \tag{11}$$

As it holds for each $i \in [k]$, this proves inequality (8).

Armed with this lemma, we are ready to present our first theorem and analyze the loss of profits by merging in markets with concave demand. The theorem is an immediate consequence of Lemma 6.

Theorem 1. *Consider a market with a concave demand function and $n \geq 3$ firms, each having a convex cost function. When k of the firms merge, for $k < n$,* *they may lose in equilibrium at most a fraction of* $1 - 1/k$ *of their total pre-merger equilibrium profits. The bound is asymptotically tight. That is, for every* $n > k \geq 2$*, there exists such a market with* n *firms, in which* k *of them merge, and lose a fraction of* $(1-1/k)\cdot(1-o(1))$ *of their total pre-merger profits, where the asymptotic notation is a function of* n*.*

Proof. We first note that the merging firms can lose at most $1 - 1/k$ of their profits by colluding, which is an immediate consequence of Lemma 6. Assume that $F_1, \ldots, F_k$ merge. By Lemma 2, the market in its state after merging also maintains Assumption 1 and Assumption 2. So, by Lemma 1, the equilibrium in the market before merging exists and is unique, and so is the equilibrium in it afterwards.

By Lemma 6, the profit of the merged firm in equilibrium is at least the profit of F_i in equilibrium, for every $i \in [k]$, before they merged. Thus:

$$\widetilde{\Pi}_{1,\ldots,k} \geq \ max\left\{\Pi_1, \Pi_2, \ldots, \Pi_k\right\} \geq \frac{1}{k} \cdot (\Pi_1 + \Pi_2 + \ldots + \Pi_k) \tag{12}$$

So the merging firms indeed lose a fraction of $1 - \frac{\widetilde{\Pi}_{1,\ldots,k}}{\Pi_1+\Pi_2+\ldots+\Pi_k} \leq 1 - \frac{1}{k}$ of their total pre-merger profits as a consequence of that merging.

Regarding the tightness, as those examples occur even in markets with affine demand functions, they are presented in Proposition 2 in the next section. In those examples, the k firms lose a fraction of exactly $max\{1 - \frac{(n+1)^2}{k(n+2-k)^2}, 0\} = \left(1 - \frac{1}{k}\right) \cdot (1 - o(1))$ of their total pre-merger profits.

We mention that when the number of merging firms is $\Theta(n)$, rather than a fixed constant (n is the total number of firms) – the merging firms may lose an arbitrarily high fraction of their profits. This is formally shown in Section 4.

We stress that there are examples that realize the bound from Theorem 1, up to an arbitrarily small constant, even in markets consisting of a small number of firms, i.e., not only asymptotically. These examples exhibit the potential severity of the merger paradox in general markets, even when all firms have simple linear cost functions. The full statement of the corresponding proposition and its proof can be found in the full version of our paper.

4 Markets with Affine Demand

This section presents our main theorem, improving the bound from Theorem 1 for the case of affine demand functions. As earlier mentioned, the loss of two merging firms can be substantial in general. However, in this section we show that when the demand function is affine this loss is mild; in such markets, two merging firms will always lose at most 1/9 of their pre-merger profits, with any profile of convex cost functions. The tightness of the bound and extensions to larger markets are also discussed.

Our main result of this paper is the following:

Theorem 2. *Consider a market with an affine demand function and three firms with convex cost functions. When two of the firms merge, they may lose at most a fraction of* $1/9$ *of their total pre-merger profits in equilibrium. Moreover, there exists such a market, in which the two merging firms lose exactly a fraction of* $1/9$ *of their total profits by merging.*

4.1 Warm Up – Affine Demand, Linear Costs

This subsection is concerned with a subset of the above markets, namely those in which the cost functions are all linear. The aforementioned tight bound is proved for those markets. The proof for general markets, given in the following subsection, is obtained via a reduction to the analysis of markets with linear costs, and specifically to a variant of the analysis from the current subsection.

The following proposition presents a tight bound of $1-\frac{(n+1)^2}{2n^2}$ on the fraction of profit losses of two firms out of $n \geq 3$ in markets with linear costs. For simplicity, this proposition focuses on the internal equilibria case. Its proof can be found in the full version of our paper.

Proposition 1. *Consider a market with an affine demand function and* $n \geq 3$ *firms with linear cost functions, in which two of the firms merge. Assume that all firms produce positive quantities in that market, pre-merging and post-merging. Then, the merging firms may lose at most a fraction of* $1-\frac{(n+1)^2}{2n^2}$ *of their total pre-merger profits in equilibrium. Moreover, for every* $n \geq 3$*, there exists such a market, in which the merging firms lose exactly a fraction of* $1-\frac{(n+1)^2}{2n^2}$ *of their total profits by merging.*

Similarly, it can be shown that in the setting of Proposition 1, whenever the merging firms have the same cost function, excluding some degenerate cases, the loss of profits is exactly $1-\frac{(n+1)^2}{2n^2}$. This is regardless of the market share of the merging firms, compared to that of the non-merging firms. The full statement of this observation and its proof can also be found in the full version of our paper.

4.2 Main Result

In order to handle the case of general cost functions, as earlier stressed, we reduce the problem to the analysis of a simpler market, namely a market with linear cost functions. Note that one might intuitively argue that linear markets would straightforwardly form the worst-case example in terms of losses due to merging, since in the symmetric case, there are no cost synergies. That is, in this case, the merged firm does not improve its cost function as compared to the cost functions of the firms that constitute it. However, mergers in the linear case could potentially lead to a higher increase in prices, as compared to the general case, which balances the former effect.

As previously emphasized, our focus is on the case of two merging firms out of three. The following lemma shows a reduction from the analysis of the general case, to the case in which the *non-merging* firm has a linear cost function. We show that for every market with general convex cost functions in which two firms merge, there exists a (possibly different) market such that the non-merging firm has a linear cost function, and in which the two merging firms attain a *higher* fraction of loss.

Lemma 7. *Consider a market with an affine demand function and three firms with convex cost functions. Assume that when two of them merge, they lose a fraction of η of their total pre-merger profits. Then, there exists a linear function, such that replacing the cost functions of the non-merging firm by it, yields a market in which the merging firms lose a fraction of at least η of their total pre-merger profits due to merging.*

Proof. Assume that F_1 and F_2 merge, and denote the original market by M. By Lemma 2, M in its state after merging maintains Assumption 1 and Assumption 2, similarly to its pre-merger state. So, by Lemma 1, the Cournot equilibrium in M before F_1 and F_2 merge exists and is unique, and so is the Cournot equilibrium in M afterwards.

Denote by LIN_1 the market obtained from M by replacing $C_3^M(\cdot)$ with the linear function $C_3^{\mathrm{LIN}_1}(x) = c_3 \cdot x$, where $c_3 = C_3'^M(x_3^M)$. By Lemma 1, x^M is also a unique Cournot equilibrium in the market LIN_1, since it preserves the necessary and sufficient conditions in the lemma. Since the cost functions of F_1 and F_2 in LIN_1 are identical to those of M, the profits of these two firms are equal in both markets.

$$\Pi_1^M + \Pi_2^M = \Pi_1^{\mathrm{LIN}_1} + \Pi_2^{\mathrm{LIN}_1} \tag{13}$$

Similarly, denote by LIN_2 the market obtained from M by replacing $C_3^M(\cdot)$ with the linear function $C_3^{\mathrm{LIN}_2}(x) = \widetilde{c}_3 \cdot x$, where $\widetilde{c}_3 = \widetilde{C}_3'^M(\widetilde{x}_3^M)$ (recall that $\widetilde{x}_3^M$ is the quantity F_3 produces in equilibrium after F_1 and F_2 merged). By the same reasoning as above, $\widetilde{x}^M$ is also the unique post-merger Cournot equilibrium in the market LIN_2. Similarly to the above argument, the profit of the merged firm is equal in both markets. That is,

$$\widetilde{\Pi}_{1,2}^M = \widetilde{\Pi}_{1,2}^{\mathrm{LIN}_2} \tag{14}$$

Now, Lemma 3 assures that $\widetilde{x}_3^M \geq x_3^M$, and since C_3^M is convex, i.e., its slope is increasing in its input, it follows that $\widetilde{c}_3 \geq c_3$. Therefore, applying Lemma 4 on LIN_1 and LIN_2 results in

$$\Pi_1^{\mathrm{LIN}_2} + \Pi_2^{\mathrm{LIN}_2} \geq \Pi_1^{\mathrm{LIN}_1} + \Pi_2^{\mathrm{LIN}_1} \tag{15}$$

Thus, F_1 and F_2 lose the following fraction of their profits by merging in the market LIN_2:

$$1 - \frac{\widetilde{\Pi}_{1,2}^{\text{LIN}_2}}{\Pi_1^{\text{LIN}_2} + \Pi_2^{\text{LIN}_2}} \geq 1 - \frac{\widetilde{\Pi}_{1,2}^{\text{LIN}_2}}{\Pi_1^{\text{LIN}_1} + \Pi_2^{\text{LIN}_1}} = \tag{16}$$

$$= 1 - \frac{\widetilde{\Pi}_{1,2}^{M}}{\Pi_1^{\text{LIN}_1} + \Pi_2^{\text{LIN}_1}} = \tag{17}$$

$$= 1 - \frac{\widetilde{\Pi}_{1,2}^{M}}{\Pi_1^{M} + \Pi_2^{M}} = \eta \tag{18}$$

The inequality follows from (15), the first equality follows from (14) and the second from (13). Therefore, the statement in the lemma indeed holds.

We turn to prove **Theorem 2**, the main result of our paper:

Proof. We show first that two merging firms lose at most a fraction of 1/9 of their profits in any such market. Denote that market by M and its inverse demand function by $P(X) = b - a \cdot X$ for some $a, b > 0$. Assume that F_1 and F_2 merge. By Lemma 2, the cost function of the merged firm adheres to Assumption 1. Therefore, M maintains Assumption 1 and Assumption 2 in both of its states, pre-merger and post-merger, and by Lemma 1, it has a unique Cournot equilibrium x^M before F_1 and F_2 merge, and a unique Cournot equilibrium $\widetilde{x}^M$ afterwards.

Assume, for now, that x^M and $\widetilde{x}^M$ are both are internal Cournot equilibria, an assumption we later relax. Assume w.l.o.g. that the non-merging firm F_3 has a linear cost function, i.e., $C_3^M(x) = c_3 \cdot x$ for some $c_3 \geq 0$. We can safely assume that, since Lemma 7 guarantees that if it has a non-linear cost function, replacing it by some specific linear function yields a market in which F_1 and F_2 lose a (weakly) higher fraction of profits by merging, and we can analyze the latter.

Our objective is showing that $\widetilde{\Pi}_{1,2}^M \geq \frac{8}{9} \cdot \left(\Pi_1^M + \Pi_2^M\right)$.

Step 1: First, we express $\widetilde{\Pi}_{1,2}^M$ as the profit of a (merged) firm in a market in which *all* firms have linear cost functions, plus some non-negative value defined later. Concretely, consider the market LIN_1 obtained from M by replacing the functions $C_1^M(x)$ and $C_2^M(x)$ by the function $C^{\text{LIN}_1}(x) = \widetilde{c} \cdot x$, where $\widetilde{c} = \widetilde{C}_{1,2}'^M\left(\widetilde{x}_{1,2}^M\right)$. Namely, $\widetilde{c}$ is the slope of the line tangent to the cost function of the merged firm, at the point it produces in equilibrium. Note that $\widetilde{C}_{1,2}^{\text{LIN}_1}(\cdot) \equiv C^{\text{LIN}_1}(\cdot)$, from the definition of the cost function of a merged firm given in Equation (3). To see this, note that from linearity:

$$\forall x, x' \geq 0 \quad C^{\text{LIN}_1}(x) + C^{\text{LIN}_1}(x') = C^{\text{LIN}_1}(x + x') \tag{19}$$

Therefore, by Lemma 1, $\widetilde{x}^M$ is also a unique Cournot equilibrium in the market LIN_1 in its post-merger state, since it preserves the necessary and sufficient conditions in the lemma. Denote by $\widetilde{x}_i^M$ the amount that factory F_i produces in $\widetilde{x}^M$, for $i = 1, 2$. Since M and LIN_1 have the same unique post-merger eq., then

$$\widetilde{\Pi}^M_{1,2} = \widetilde{\Pi}^{\mathrm{LIN}_1}_{1,2} + \widetilde{C}^{\mathrm{LIN}_1}_{1,2}(\widetilde{x}^M_{1,2}) - \widetilde{C}^M_{1,2}(\widetilde{x}^M_{1,2}) = \tag{20}$$

$$= \widetilde{\Pi}^{\mathrm{LIN}_1}_{1,2} + C^{\mathrm{LIN}_1}(\widetilde{x}^M_1 + \widetilde{x}^M_2) - C^M_1(\widetilde{x}^M_1) - C^M_2(\widetilde{x}^M_2) = \tag{21}$$

$$= \widetilde{\Pi}^{\mathrm{LIN}_1}_{1,2} + \widetilde{c} \cdot \widetilde{x}^M_1 - C^M_1(\widetilde{x}^M_1) + \widetilde{c} \cdot \widetilde{x}^M_2 - C^M_2(\widetilde{x}^M_2) \tag{22}$$

The 2nd inequality is by the definition of $\widetilde{x}^M_1$ and $\widetilde{x}^M_2$, and from the mentioned fact that $\widetilde{C}^{\mathrm{LIN}_1}_{1,2}(\cdot) \equiv C^{\mathrm{LIN}_1}(\cdot)$. The 3rd is due to the linearity of $C^{\mathrm{LIN}_1}(\cdot)$.

As mentioned, we show that (22) equals $\widetilde{\Pi}^{\mathrm{LIN}_1}_{1,2}$ plus some non-negative constant, i.e., that

$$\widetilde{c} \cdot \widetilde{x}^M_1 - C^M_1(\widetilde{x}^M_1) + \widetilde{c} \cdot \widetilde{x}^M_2 - C^M_2(\widetilde{x}^M_2) \geq 0 \tag{23}$$

For $i = 1, 2$, denote $z_i = \widetilde{c} \cdot \widetilde{x}^M_i - C^M_i(\widetilde{x}^M_i)$. If $\widetilde{x}^M_i = 0$, then $z_i = 0$. Otherwise, by Lemma 2, $\widetilde{c}$ is the slope of the line tangent to C^M_i at $\widetilde{x}^M_i$.

Recall that any convex function lies above any line tangent to it. That is, if $C(\cdot)$ is convex and differentiable, then for every $x, x_0 \geq 0$:

$$C(x) \geq C'(x_0) \cdot (x - x_0) + C(x_0) \tag{24}$$

Applying Inequality (24) to the convex $C^M_i(\cdot)$ by plugging $x = 0$ and $x_0 = \widetilde{x}^M_i$ we obtain that

$$0 = C^M_i(0) \geq \widetilde{c} \cdot (-\widetilde{x}^M_i) + C^M_i(\widetilde{x}^M_i) \tag{25}$$

Putting it differently, $z_i \geq 0$ for $i = 1, 2$, and

$$\widetilde{\Pi}^M_{1,2} = \widetilde{\Pi}^{\mathrm{LIN}_1}_{1,2} + z_1 + z_2 \tag{26}$$

Step 2: Now, we deploy another replacement of the cost functions of F_1 and F_2, similar to the one presented above. The difference is that this time, it guarantees that the *pre-merger* Cournot equilibrium in the resulting market is identical to that of M, instead of the *post-merger* one. For that, consider a different market, LIN_2, obtained from M by replacing the functions $C^M_1(x)$ and $C^M_2(x)$ by the functions $C^{\mathrm{LIN}_2}_1(x) = c_1 \cdot x$ and $C^{\mathrm{LIN}_2}_2(x) = c_2 \cdot x$ respectively, where $c_i = C'^M_i(x^M_i)$ for $i = 1, 2$. Note that x^M is also a unique Cournot equilibrium in the market LIN_2, as it preserves the necessary and sufficient conditions given in Lemma 1. Plugging $x = x^M_1$ and $x_0 = \widetilde{x}^M_1$ in Inequality (24) yields

$$C^M_i(x^M_i) \geq \widetilde{c} \cdot (x^M_i - \widetilde{x}^M_i) + C^M_i(\widetilde{x}^M_i) = \widetilde{c} \cdot x^M_i - z_i \tag{27}$$

So, for $i = 1, 2$:

$$\Pi^M_i(x^M) = \Pi^{\mathrm{LIN}_1}_i(x^M) + C^{\mathrm{LIN}_1}(x^M_i) - C^M_i(x^M_i) = \tag{28}$$

$$= \Pi^{\mathrm{LIN}_1}_i(x^{\mathrm{LIN}_2}) + C^{\mathrm{LIN}_1}(x^M_i) - C^M_i(x^M_i) = \tag{29}$$

$$= \Pi^{\mathrm{LIN}_1}_i(x^{\mathrm{LIN}_2}) + \widetilde{c} \cdot x^M_i - C^M_i(x^M_i) \leq \tag{30}$$

$$\leq \Pi^{\mathrm{LIN}_1}_i(x^{\mathrm{LIN}_2}) + z_i \tag{31}$$

The first equality follows from the definition of LIN_1 and the profit functions. The second from $x^M = x^{\text{LIN}_2}$ as we just mentioned. The third from the definition of C^{LIN_1}, and the inequality follows from Inequality (27).

Step 3: The computations in Step 1 and Step 2 imply that

$$\frac{\widetilde{\Pi}^M_{1,2}}{\Pi^M_1 + \Pi^M_2} = \frac{\widetilde{\Pi}^{\text{LIN}_1}_{1,2} + z_1 + z_2}{\Pi^M_1 + \Pi^M_2} \tag{32}$$

$$\geq \frac{\widetilde{\Pi}^{\text{LIN}_1}_{1,2} + z_1 + z_2}{\Pi^{\text{LIN}_1}_1(x^{\text{LIN}_2}) + \Pi^{\text{LIN}_1}_2(x^{\text{LIN}_2}) + z_1 + z_2} \tag{33}$$

where the equality follows from Eq. (26) and the inequality from Eq. (31).

Note that since we regularly assume that $\Pi^M_1 + \Pi^M_2 > 0$ (as otherwise the statement in the theorem becomes trivial), then Inequality (31) also implies that the denominator in (33) is positive. Denote the numerator of (33) by $x \geq 0$ and the denominator by $y \geq 0$. If $x/y \geq 1$ then we are done, as there are no losses due to merging in that case. In particular, if $y - z_1 - z_2 < 0$, that ratio is at least 1, as $x - z_1 - z_2 = \widetilde{\Pi}^{\text{LIN}_1}_{1,2} \geq 0 > y - z_1 - z_2$.

So, assume that $y - z_1 - z_2 \geq 0$ and that $y > x$. Note that in general, $y - z_1 - z_2 > x - z_1 - z_2 \geq 0$ and $z_1 + z_2 \geq 0$ imply that $\frac{x}{y} \geq \frac{x - z_1 - z_2}{y - z_1 - z_2}$.

Therefore the term in (33) is at least $\frac{\widetilde{\Pi}^{\text{LIN}_1}_{1,2}}{\Pi^{\text{LIN}_1}_1(x^{\text{LIN}_2}) + \Pi^{\text{LIN}_1}_2(x^{\text{LIN}_2})}$. This ratio presents a scenario which is a variant of Proposition 1 from Section 4 regarding the ratio of profits in markets with linear cost functions. This variant concerns, this time, hybrid linear markets that take into account only the equilibrium productions for the post-merger state, ignoring those of the pre-merger state. In the full version of our papers, we fully analyze this type of hybrid markets, and the ratio is shown to be at least 8/9 by that lemma, as required.

So, we conclude that the statement holds for the internal equilibria case. The treatment of the corner cases in fully discussed in the full version of our paper. Note that the tightness of the bound immediately follows from Proposition 1, by plugging $n = 3$, so we are done.

4.3 Arbitrarily High Losses Due to Merging

In the previous subsections, we gave bounds on the fraction of loss of two merging firms. This subsection discusses the case of a larger number of merging firms, i.e., more than two, and shows that when the total number of firms tends to infinity, the fraction of loss due to merging approaches 1, even in simple markets with affine demand functions and linear cost functions. This also covers the cliffhanger from the previous section, namely the tightness of the statement in Theorem 1.

This is formally stated and proved in the following proposition, which generalizes the tightness proof in Proposition 1, and in the corollary that follows. Note that the closed form representation of the production levels and the profits in markets with linear costs used below, can be found and are proved in the full version of our paper.

Proposition 2. *For every $n > k \geq 2$, there exists a market with n firms, in which k of them merge, and consequently lose a fraction of $max\{1-\frac{(n+1)^2}{k(n+2-k)^2}, 0\}$ of their pre-merger profits.*

Proof. Take M to have the inverse demand function $P(X) = 1 - X$ with n identical firms, each having the cost function $C(\cdot) \equiv 0$. The candidate for the pre-merger production level of each of the firms is

$$x_i = \frac{1 - (n+1) \cdot 0 + \sum_{j=1}^{n} 0}{(n+1) \cdot 1} = \frac{1}{n+1} > 0, \quad \text{for all } i \in [n] \tag{34}$$

As for the market in its post-merger state, note that by the definition in Equation (3), it holds that $\widetilde{C}_{1,\ldots,k}(\cdot) \equiv C_k(\cdot)$, as for all $x_1, \ldots, x_k \geq 0$:

$$C_1(x_1) + \ldots + C_k(x_k) = C_k(x_1) + \ldots + C_k(x_k) = C_k(x_1 + \ldots + x_k) \tag{35}$$

The first equality follows from the fact that the firms are identical, and the second equality follows from the linearity of C_k.

So, the analysis of M in its post-merger state is identical to the analysis of M, with $n - k + 1$ firms instead of n. Thus, the candidate for the post-merger production level of each firm is (for all $i \in [n] \setminus [k]$):

$$\widetilde{x}_{1,\ldots,k} = x_i = \frac{1 - (n+2-k) \cdot 0 + \sum_{j=2}^{n} 0}{(n+2-k) \cdot 1} = \frac{1}{(n+2-k)} > 0 \tag{36}$$

Since those candidates are all positive, those are the actual production levels in the two equilibria. In addition,

$$\Pi_i = 1 \cdot \left(x_i^M\right)^2 = 1 \cdot \left(\frac{1}{n+1}\right)^2, \text{ for all } i \in [n] \setminus [k] \tag{37}$$

$$\widetilde{\Pi}_{1,\ldots,k} = 1 \cdot \left(\widetilde{x}_{1,\ldots,k}^M\right)^2 = 1 \cdot \left(\frac{1}{n+2-k}\right)^2 \tag{38}$$

This implies that $\frac{\widetilde{\Pi}_{1,\ldots,k}}{\Pi_1 + \Pi_2 + \ldots + \Pi_k} = \frac{(n+1)^2}{k(n+2-k)^2}$, as required.

Corollary 1. *For every $\epsilon > 0$, there exist $n > k \geq 2$ and a market with n firms, such that when k of them merge, they lose a fraction of $1 - \epsilon$ of their total pre-merger profits.*

Proof. Let $\epsilon > 0$. Fix some even $n > max\{\frac{32}{\epsilon}, 4\}$ and $k = \frac{1}{2}n$.

By the previous proposition, there exists a market with n firms, in which k of them merge, and lose the following fraction of their pre-merger profits:

$$1 - \frac{(n+1)^2}{k(n+2-k)^2} = 1 - \frac{(n+1)^2}{\frac{1}{2}n \cdot (n+2-\frac{1}{2}n)^2} = \tag{39}$$

$$= 1 - \frac{8(n+1)^2}{n(n+4)^2} \geq \tag{40}$$

$$\geq 1 - \frac{8(2n)^2}{n \cdot n^2} = 1 - \frac{32}{n} \geq 1 - \epsilon \tag{41}$$

and the result holds.

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